and

$$\theta = (2\lambda - 1)\frac{\pi}{2} + \frac{q}{p}(4\mu \pm 1)\frac{\pi}{2}, \qquad \left(\lambda = 1, 2, 3, \cdots, \frac{q-1}{2};\ \mu = 0, 1, 2, \cdots, p-1\right);$$

determine the same set of points on the curve

$$\rho = a \cos\frac{p}{q}\theta,$$

where p and q are two odd integers without a common factor, and a is any constant.

471. Proposed by C. N. SCHMALL, New York City.

In the ellipse $x^2/a^2 + y^2/b^2 = 1$, an *equilateral* hexagon is inscribed with two sides parallel to the major axis. In the major auxiliary circle the same thing is done. If H_1 and H_2 be the sides of the hexagons, and e the eccentricity of the ellipse, show that $H_1 : H_2 :: 4 - 2e^2 : 4 - e^2$

CALCULUS.

390. Proposed by WILSON L. MISER, University of Minnesota.

Show that the triangle whose area is a constant and whose perimeter is a minimum is equilateral.

391. Proposed by H. B. PHILLIPS, Massachusetts Institute of Technology.

If $0 < \lambda < 1$ and $0 < x < \pi$, show that the function $(\sin \lambda x)/(\sin x)$ increases as x increases.

392. Proposed by HORACE OLSON, Student at The University of Chicago.

Two right circular cylinders of radii a and b respectively, are placed so that their axes intersect at right angles. Find the volume common to them.

MECHANICS.

314. Proposed by C. N. SCHMALL, New York City.

A rectangular box of height h, and having a plane mirror for its bottom, contains a quantity of water of unknown height x. In the lid are two small apertures distant $2a$ from each other. A ray of light entering one aperture with an angle of incidence ϕ, emerges, after refraction and reflection, through the other aperture. If μ be the index of refraction of water, show that the height of the water is

$$x = \frac{(h \tan\phi - a)}{\left[\tan\phi - \dfrac{\sin\phi}{(\mu^2 - \sin^2\phi)^{\frac{1}{2}}}\right]}.$$

SOLUTIONS OF PROBLEMS.

ALGEBRA.

427. Proposed by CLIFFORD N. MILLS, Brookings, S. Dak.

If $r\sin(\theta + \alpha) = m$, and $r\cos(\theta + \beta) = n$, show that

$$r = \frac{\sqrt{m^2 + n^2 - 2mn\sin(\alpha - \beta)}}{\cos(\alpha - \beta)}.$$

SOLUTION BY J. H. KELLOGG, Oberlin College.

From trigonometry, using only the positive square root, we know that

$$\sin(A+B) = \sqrt{(\sin A\cos B + \cos A\sin B)^2}.$$

$$= \sqrt{\sin^2 A(1-\sin^2 B) + (1-\sin^2 A)\sin^2 B + 2\sin A\cos A\sin B\cos B}$$

$$= \sqrt{\sin^2 A + \sin^2 B + 2\sin A\sin B(\cos A\cos B - \sin A\sin B)}$$

$$= \sqrt{\sin^2 A + \sin^2 B + 2\sin A\sin B\cos(A+B)}.$$

From the given equations we have

$$\sin(\theta+\alpha) = \frac{m}{r} \qquad \text{and} \qquad \sin\left[\frac{\pi}{2} - (\theta+\beta)\right] = \frac{n}{r}.$$

Hence letting

$$A = (\theta+\alpha) \qquad \text{and} \qquad B = \left[\frac{\pi}{2} - (\theta+\beta)\right],$$

we have

$$(A+B) = \left[\frac{\pi}{2} + (\alpha-\beta)\right].$$

Substituting, we have

$$\sin\left[\frac{\pi}{2} + (\alpha-\beta)\right]$$

$$= \sqrt{\sin^2(\theta+\alpha) + \sin^2\left[\frac{\pi}{2} - (\theta+\beta)\right] + 2\sin(\theta+\alpha)\sin\left[\frac{\pi}{2} - (\theta+\beta)\right]\cos\left[\frac{\pi}{2} + (\alpha-\beta\right]},$$

or

$$\cos(\alpha-\beta) = \sqrt{\frac{m^2}{r^2} + \frac{n^2}{r^2} - 2\frac{m}{r}\frac{n}{r}\sin(\alpha-\beta)}.$$

Solving for r, we have

$$r = \frac{\sqrt{m^2+n^2-2mn\sin(\alpha-\beta)}}{\cos(\alpha-\beta)}.$$

Also solved by B. J. Brown, Herbert N. Carlton, Nathan Altshiller, C. E. Horne, C. N. Schmall, Paul Capron, J. A. Caparo, Frank Irwin, Frank R. Morris, S. A. Joffe, V. M. Spunar, L. G. Weld, George W. Hartwell, Elijah Swift, A. W. Smith, Joseph B. Reynolds, Richard Morris, Walter C. Eells, Albert N. Nauer, Elmer Schuyler, and A. M. Harding.

428. Proposed by FRANK IRWIN, University of California.

If the roots of the equation

$$x^n - na_1x^{n-1} + \binom{n}{2}a_2x^{n-2} + \cdots = 0$$

are all real, the condition that they should all be equal is $a_1^2 = a_2$. A proof of the sufficiency of the condition is readily obtained from a consideration of derivatives. A proof is desired not based on such considerations.

Solution by Elijah Swift, University of Vermont.

We are to show that if the roots are all real, and if $a_1^2 = a_2$, the roots are all equal. For that purpose write the above equation in the form